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The Quantum Speed up as Advanced Cognition of the Solution

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Abstract Solving a problem requires a problem solving step (deriving, from the formulation of the problem, the solution algorithm) and a computation step (running the algorithm). The latter step is generally oblivious of the former. We unify the two steps into a single physical interaction: a many body interaction in an idealized classical framework, a measurement interaction in the quantum framework. The many body interaction is a useful conceptual reference. The coordinates of the moving parts of a perfect machine are submitted to a relation representing problem-solution interdependence. Moving an "input" part nondeterministically produces a solution through a many body interaction. The kinematics and the statistics of this problem solving mechanism apply to quantum computation—once the physical representation is extended to the oracle that produces the problem. Configuration space is replaced by phase space. The relation between the coordinates of the machine parts now applies to a set of variables representing the populations of the qubits of a quantum register during reduction. The many body interaction is replaced by the measurement interaction, which changes the population variables from the values before to the values after measurement (and the forward evolution into the backward evolution, the same unitary transformation but ending with the state after measurement). Quantum computation is *reduction* on the solution of the problem under the problem-solution interdependence relation.

The speed up is explained by a simple consideration of time-symmetry, it is the gain of information about the solution due to backdating, to before running the algorithm, a time-symmetric part of the reduction on the solution. This advanced cognition of the solution reduces the solution space to be explored by the algorithm. The quantum algorithm takes the time taken by a classical algorithm that knows in advance 50% of the information acquired by reading the solution (i.e. by measuring the content of the computer register at the end of the quantum algorithm).

From another standpoint, the notion that a computation process is condensed into a single physical interaction explains the fact that we perceive many things at the same time in the introspective "present" (the instant of the interaction in the classical case, the time interval spanned by backdated reduction in the quantum case).

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1 Introduction

Solving a problem requires two steps. A problem solving step (deriving, from the formulation of the problem, the solution algorithm) and a computation step (running the algorithm). The latter step is generally oblivious of the former. We unify the two steps into a single physical interaction, as follows:

1. For the conventional representation of computation, we adopt the so called "circuit model". At the logical level, a set of Boolean variables (representing the computer register) undergoes a sequence of elementary reversible/deterministic input output transformations, changing input values into output values. Through this sequence, an overall input is transformed into an overall output representing the solution of the problem. The sequence can be represented as a *linear Boolean network*, a series of reversible Boolean gates with no feedback loops and no preassigned values on the outputs, where the input logically propagates to the output in a deterministic way. At the physical level, a computer register undergoes a sequence of deterministic transformations (corresponding to the logical ones) that change input states of the register into output states until the overall output is reached. Reading the register content in the overall output state yields the solution of the problem. These features are common to classical computation and to the unitary evolution stage of quantum computation. This representation of computation is oblivious of the problem (in the quantum algorithms, of the oracle that produces the problem), an omission that makes the speed up conceptually unexplained.
2. Computation is unified with problem solving as follows. We consider the *non-linear Boolean network* representing the original problem, to be distinguished from the above said linear network, which represents the algorithm that solves the problem. The nonlinear network is generally exponentially smaller, has logically irreversible gates or partial gates, feedback loops (outputs feeding back into inputs), and outputs with preassigned values. Solving this network means finding a Boolean assignment that satisfies all network elements (gates, wires, preassigned Boolean values). In general the non linear network is not directly solvable by the deterministic propagation of an input into an output; it is transformed into an exponentially larger linear network (representing the solution algorithm), solvable by this kind of propagation. Here we proceed in a different way, identifying a physical interaction that directly produces the solutions of the nonlinear network. We replace the Boolean variables x_i of the nonlinear network by ratios between real non-negative variables X_i/Q, the network constraints by equations on these ratios. For $Q > 0$, the solutions of this system of equations correspond to the solutions of the nonlinear Boolean network. The relation between variables (the ratios) established by the system of equations represents problem-solution interdependence. We provide two ways of physically producing the solutions of the system of equations: by a classical many body interaction and by a quantum measurement interaction. The many body interaction is inspired to a well known paradox of classical mechanics. Statically, the application of external forces to a perfectly rigid body is balanced by infinitely many distributions of stress inside the body, against one distribution if the body is flexible. This paradox is ported to a perfectly rigid body made of moving parts, whose coordinates X_i and Q are submitted to mechanical constraints representing the system of equations. In the initial

configuration all coordinates are zero. By applying a force to the input part of coordinate Q, the many distributions of stress inside the body, find a combination of movements of the machine parts that satisfies all the constraints at the same time. In more detail, pushing the input part from $Q = 0$ to $Q > 0$, brings in the many body problem, the non-determination of the dynamics. By applying the principle of sufficient reason, we postulate a many body interaction that produces a solution of the problem with probability proportional to the mass of the machine parts moved to produce that solution. The kinematics and the statistics of this idealized classical interaction can represent a quantum measurement interaction. The "coordinates" of the machine parts X_i/Q are replaced by a set of variables representing the populations of (the reduced density operators of) the qubits of a quantum register immediately before and after measurement. The measurement interaction changes these population variables from the values immediately before to the values immediately after measurement. The problem-solution interdependence relation linearly extends to an infinite set of variables representing the amplitudes of the computational basis vectors throughout the quantum process. Under the extended relation, the measurement interaction changes the forward evolution (the unitary evolution ending with the state before measurement) into the backward evolution (the same unitary transformation but ending with the state after measurement). In this "relational" representation, quantum computation is reduction on the solution under the relation representing problem-solution interdependence.

3. Applied to the quantum algorithms, this representation of problem solving and computation (for short, computation from now on) provides a unified explanation of the speed up. In the first place, the physical representation of computation must be extended to comprise the oracle that produces the problem. Then any known quantum algorithm becomes reduction on the solution under the relation representing problem-solution interdependence. The quantum speed up is explained by a simple consideration of time-symmetry. It is the gain of information about the solution of the problem due to backdating, to before running the algorithm, a time-symmetric part of the reduction on the solution. This advanced cognition of the solution[1] reduces the size of the solution space to be explored by the algorithm. The quantum algorithm takes the time taken by a classical algorithm that knows in advance 50% of the information acquired by reading the solution (i.e. by measuring the content of the computer register at the end of the quantum algorithm). This is verified for the algorithms of Deutsch, Grover, and Simon (also in the case of the generalized Simon's problem, thus for the hidden subgroup problems), namely for both quadratic and exponential speed ups.
4. Nondeterminism is here a fundamental feature of computation, it is not loosing track of a propagation spreading through too many degrees of freedom. It is capability of making choices satisfying many (like in many body) constraints at the same time.
5. At least in the idealized classical analogy, nondeterminism goes along with reversibility. The machine motion is reversible (dissipationless), but not invertible. Inverting the direction of motion might not lead to run the same trajectory in the opposite way, it can change the trajectory (in occasion of the many body interaction).
6. The notion that an entire problem solving and computation process can be condensed into a single interaction (an idealized classical many body interaction or a quantum measurement interaction) can also explain the unity of perception, the fact that we perceive (assumedly, process) so many things together at the same time in the so called "present".

[1] We have chosen the term "advanced" because it is reminiscent of the "advanced wave", going backward in time, of Cramer's transactional interpretation of quantum mechanics.

Tacking into account many constraints at the same time is exactly what a classical many body interaction, or a quantum measurement interaction, does. The physical representation of the introspective notion of "present" is the instant of the interaction in the classical case, the time interval spanned by backdated reduction in the quantum case.

2 The Notion of Perfect Relation in Classical and Quantum Physics

The idea that many things are processed all together at the same time, standing at the basis of relational computation, is formalized by resorting to a notion of the Gestalt theory (e.g. [18]). The wholeness/unity of a physical situation implies that there is a relation—a "simultaneous dependence" in the language of the theory—between all the quantitative variables describing it.

An example of such a relation in classical physics is "force equal mass times acceleration" in the case of a point mass. In view of what will follow, it should be noted that this relation is implicitly assumed to be objectively perfect. If we see it as a mechanism, whose degrees of freedom are the variables related by the law, this mechanism should be perfectly accurate, rigid, and reversible—it is not the case that Newton's second law gets deformed because of flexibility or jams because of friction or irregularities.

Another important feature of the relations that we find in Nature, is that they can be nonfunctional, which is also the case of Newton's law. The change of any one variable is correlated with an identical change of the product or ratio of the other two variables but does not determine their individual changes. Correspondingly, Newton's law can host nondeterminism in the form of the many body problem.

The requirement that the relation representing problem-solution interdependence is perfect, is essential in the case of the classical many body interaction, it is absorbed into the quantum principle and the notion of qubit [10] in the case of the measurement interaction. That infinite classical precision can be dispensed for because of quantization has already been noted by Finkelstein [11].

3 Relational Computation in the Classical Framework

We postulate a many body interaction inspired to a well known paradox of classical mechanics: statically, the application of external forces to a perfectly rigid body is balanced by infinitely many distributions of stress inside the body, against one distribution if the body is flexible. This paradox is ported to a perfectly rigid body made of moving parts, whose coordinates are submitted to mechanical constraints representing the problem. By applying a force to an input part, the many distributions of stress inside the body find a combination of movements of the body's parts that satisfies all the constraints at the same time.

It is interesting to note that giving up the limitation to two body interaction marks the departure from classical computation. The fundamental physical model of classical computation is the bouncing ball model of reversible computation [12]. The variables at stake are ball positions and momenta. Outside collisions, there is no simultaneous dependence between the variables of different balls, which are independent of each other. During collision, there is simultaneous dependence between the variables of the colliding balls, but this is confined to ball pairs (there can be several collisions at the same time, but involving independent ball pairs, with no simultaneous dependence between the variables of different pairs). The simultaneous collision between many balls is avoided to avoid the many body problem, the non-determination of the dynamics.

Instead, by assuming a perfect simultaneous dependence between all computational variables, one can devise an idealized classical machine that, thanks to a many body interaction, nondeterministically produces the solution of a (either linear or non linear) system of Boolean equations under the simultaneous influence of all equations.

Let us start with the simple problem of finding the solutions ($x = 0$, $y = 1$ and $x = 1$, $y = 0$) of the single Boolean equation $y = \overline{x}$. The problem solving/computation mechanism that produces these solutions can easily be extended to any system of Boolean equations. Let Q, X, and Y be real non negative variables. The Boolean problem is transformed into the problem of finding the solutions, for $Q > 0$, of the system of equations

$$Q = X + Y, \tag{1}$$

$$Q^2 = X^2 + Y^2. \tag{2}$$

$Q = 0$ implies $X = Y = 0$, while $\frac{X}{Q}$ and $\frac{Y}{Q}$ are undetermined. When $Q > 0$, $\frac{X}{Q}$ coincides with the Boolean variable x and $\frac{Y}{Q}$ with $y = \overline{x}$. Equations (1) and (2), representing the problem constraint $y = \overline{x}$, establish a nonfunctional relation, a simultaneous dependence, between the variables Q, X, and Y. The solutions are produced under this relation by a many body interaction as follows. We put a differential gear between coordinate Q (the input of the gear) and coordinates X and Y (the two outputs of the gear), which implements (1). We put another differential gear between the squares of these coordinates, namely between the auxiliary coordinates Q', X', and Y' connected through parabolic cams to Q, X, and Y, so that $Q' = Q^2$, $X' = X^2$, and $Y' = Y^2$, which implements (2).

The initial machine configuration is $Q = X = Y = 0$; it can be argued that any motion of the part of coordinate Q from $Q = 0$ to $Q > 0$ instantly produces a solution in a nondeterministic way, as follows. The many body problem is the problem of the non-determination of the dynamics in the case of perfect coincidence between interaction times of many bodies – which is the case if we try and push part Q out of $Q = 0$. Here we postulate a solution to the many body problem by applying the principle of sufficient reason. The motion of part Q could be obtained by applying a force to it. In fact, there is no reason for either X or Y, in a mutually exclusive way, not to move with Q, since either movement offers zero static resistance to the force—there is only the inertia of the machine parts.

By playing with inertia, we can also tune the probabilities that X and Y move. We require that, on average (over an ensemble of repetitions of the interaction), there is equipartition of energy among the machine degrees of freedom. Since the force applied to part Q works against the inertia of all the parts that move with Q, this implies that the probability of each solution is proportional to the mass of the parts that move to produce that solution (either Q and X or Q and Y). Under this assumption, and by assuming for example even masses for X and Y, the values of the coordinate ratios change from $\frac{X}{Q} = \frac{Y}{Q} = \frac{1}{2}$ before interaction to $\frac{X}{Q} = 1$ and $\frac{Y}{Q} = 0$ or (in a mutually exclusive way) $\frac{X}{Q} = 0$ and $\frac{Y}{Q} = 1$ after interaction.

We should note that, unlike deterministic reversible processes, the present process is not invertible. For example, we can think of connecting the input part to an ideal spring charged when $Q = 0$. On the one side, there would be oscillations without dissipation. On the other, at each oscillation, the movement of the input part from $Q = 0$ to $Q > 0$ would randomly drag either X or Y in a mutually exclusive way.

This computation mechanism is easily extended to solve any system of Boolean equations, namely (without loss of generality) a network of n partial OR gates $\mathrm{POR}(x_{i,1}, x_{i,2}, x_{i,3}) = 1$ $(i = 1, \ldots, n)$ and m wires $x_{i,j} = x_{h,k}$ (for m assignments of i, j, h, k). The truth

table of the partial OR gate i is given in the right side of table (3).

part	$x_{i,1}$	$x_{i,2}$	$x_{i,3}$
$X_{i,1}$	0	1	1
$X_{i,2}$	1	0	1
$X_{i,3}$	1	1	0

(3)

The problem solving machine for this network is defined as follows. For all i, each row j ($j = 1, 2, 3$) of the truth table is associated with a mechanical part of coordinate $X_{i,j}$ (table 3)—we also say that part $X_{i,j}$ is labelled by the values of the Boolean variables $x_{i,1}, x_{i,2}, x_{i,3}$ appearing in the corresponding row. Q and Q^2 (with a parabolic cam in between) are now the inputs of n pairs of differential gears, one linear and the other nonlinear, as before. This time, each linear gear i has three outputs of coordinates $X_{i,j}$ ($j = 1, 2, 3$), each nonlinear gears their squares (with parabolic cams in between), so that, for all i: $Q = \sum_{j=1}^{3} X_{i,j}$ and $Q^2 = \sum_{j=1}^{3} X_{i,j}^2$. Therefore, the motions of the parts of each *triplet* ($X_{i,1}$, $X_{i,2}$, $X_{i,3}$) are mutually exclusive with one another. If part $X_{i,j}$ moves with Q, we understand that $x_{i,1}, x_{i,2}, x_{i,3}$ assume the values appearing in the corresponding row.

This is justified by the following implementation of the wires. For example, let us assume that $x_{i,1} = x_{h,2}$, which means either $x_{i,1} = x_{h,2} = 0$ or $x_{i,1} = x_{h,2} = 1$. Looking at table (3), one can see that this wire must be represented by the equations $X_{i,1} = X_{h,2}$ and $X_{i,2} + X_{i,3} = X_{h,1} + X_{h,3}$. In fact, if the part that moves in the first triplet is $X_{i,1}$, this implies $x_{i,1} = 0$ (see the intersection between first row and first column of table (3)). Then, to satisfy the wire, the part that moves in the second triplet must be $X_{h,2}$, so that also $x_{h,2} = 0$ (intersection between second row and second column of table (3)—having replaced the subfix i by h). This justifies the first equation. If instead the part that moves in the first triplet is either $X_{i,2}$ or $X_{i,3}$, this implies $x_{i,1} = 1$ (intersection between second or third row and first column). Then, to satisfy the wire, the part that moves in the second triplet must be either $X_{h,1}$ or $X_{h,3}$, so that also $x_{h,2} = 1$ (intersection between first or third row and second column). This justifies the second equation. In general, we require that the sums $\sum_j X_{i,j}$, with j running over the labels with the same value of the same Boolean variable, are conserved across different triplets (e.g. we understand that $x_{i,1} = x_{h,2}$ and $x_{h,2}$ are the same Boolean variable in triplets i and h). These linear equations (representing the wires) are implemented by systems of gears between the parts involved.

At this point the thought machine is completed. By applying a force to the input part Q, the machine's motion from $Q = 0$ to $Q > 0$ produces a solution[2] under the simultaneous influence of all the problem constraints: in each triplet, there is only one part that moves, the labels of all the parts that move make up a Boolean assignment that solves the network.

We should note that this form of computation is essentially different from the causal propagation of an input into an output. For example, it can produce two inputs such that their product is a preassigned output (the nonlinear Boolean network for this problem is the network for the multiplication of two integer numbers, with a preassigned value on the output). If this were an input-output propagation (which is not), one should say that inputs are produced with advanced cognition of the output.

[2]Provided that there is one, otherwise the machine is jammed.

4 Relational Computation in the Quantum Framework

The kinematics and the statistics of the idealized classical many body interaction can apply, as it is, to a realistic quantum computation. Considering the previous example $y = \overline{x}$, the motion from $Q = 0$ to $Q > 0$ is analogous to measuring two qubits X and Y in the entangled state $|0\rangle_X|1\rangle_Y + e^{i\delta}|1\rangle_X|0\rangle_Y$, where δ is an arbitrary (even random) phase. The configuration space of the classical machine becomes the phase space of a quantum register, the relation between the coordinates of the machine parts (the ratios $\frac{X}{Q}$, $\frac{Y}{Q}$) now applies to a set of variables representing the populations of (the reduced density operators of) the qubits of a quantum register during the measurement interaction (immediately before, during, and immediately after measurement). The correspondence between coordinates and populations is:

$$\frac{X}{Q} = x_{00} = 1 - x_{11}, \qquad \frac{Y}{Q} = y_{00} = 1 - y_{11}, \tag{4}$$

where x_{00} is the variable representing the population of qubit $|0\rangle_X\langle 0|_X$ during reduction, etc. Under this correspondence, the problem-solution interdependence relation is the same as before:

$$Q = X + Y, \tag{5}$$

$$Q^2 = X^2 + Y^2. \tag{6}$$

In the transition from $Q = 0$ to $Q > 0$, the population variables change from the values before reduction $x_{00} = x_{11} = y_{00} = y_{11} = \frac{1}{2}$ to the values after reduction, one of the two mutually exclusive sets of values $x_{00} = 0$, $x_{11} = 1$, $y_{00} = 1$, $y_{11} = 0$ and $x_{00} = 1$, $x_{11} = 0$, $y_{00} = 0$, $y_{11} = 1$.

It should be noted that simultaneous dependence (functionally) extends to the amplitudes of all the basis vectors throughout the unitary evolution stage of the quantum process—for example the process of entangling X and Y starting from a sharp preparation, say $|0\rangle_X|0\rangle_Y$. Let $|\psi, t\rangle = \sum_i \alpha_i(t)|i\rangle$, be the state of the quantum system at time t between preparation and measurement, where $|i\rangle$ is the i-th computational basis vector. At any time t, any amplitude $\alpha_i(t)$ is a function of the population variables at the time of reduction:

$$\forall t : \alpha_i(t) = f_i(x_{00}, x_{11}, y_{00}, y_{11}, t). \tag{7}$$

Under the infinite system of (4), (5), (6), and (7), reduction changes the forward evolution into the backward evolution. This is only an alternative way of representing a quantum process, suited to the present context.

It should be noted that, under (7), any two amplitudes $\alpha_i(t_1)$ and $\alpha_j(t_2)$, with $t_1 \leq t_2$, also depend from one another in a time symmetric way. In other words, it is not the case that the change (from the forward to the backward value) of the amplitude at time t_1 causes the change of the amplitude at the later time t_2; causality is mutual, like in the measurement of two entangled polarizations.

5 Relational Computation and the Quantum Algorithms

We apply relational computation to the representation of the quantum algorithms. We shall consider only the input and the output of the unitary transformation performed by the quantum algorithm, therefore no previous knowledge of the subject is required to the reader. For a complete description of the quantum algorithms, in the same notation used here, see [16].

5.1 Grover's algorithm

The problem addressed by Grover's algorithm [13] is database search. It can be seen as a game between two players with a chest of N drawers; the first player (the oracle) hides a ball in drawer number k. The second player must find where the ball is. Opening drawer x to check whether the ball is in it amounts to computing the Kroneker function $\delta(k, x)$, which is 1 if $k = x$ and 0 otherwise. Here $k \in \{0, 1\}^n$, where $n = \log_2 N$ (for simplicity, let us assume that N is a power of 2).

The value of k chosen by the first player is hardwired inside a black box that, for each input x, computes $\delta(k, x)$. The second player has to find the value of k by computing $\delta(k, x)$ for different values of x. In the classical case, to find the value of k, $\delta(k, x)$ must be computed the order of N times, in the quantum case the order of $\sqrt{N}$ times—there is a quadratic speed up.

In the latter case, instead of trying a single value of x, the second player prepares an n-qubit register X in an even superposition of all the possible values of x, and computes $\delta(k, x)$ in quantum parallelism. For example, with $N = 4$ and $k = 01$, the algorithm unitarily changes the input state

$$\frac{1}{2\sqrt{2}}(|00\rangle_X + |01\rangle_X + |10\rangle_X + |11\rangle_X)(|0\rangle_V - |1\rangle_V), \tag{8}$$

into the output state

$$\frac{1}{\sqrt{2}}|01\rangle_X(|0\rangle_V - |1\rangle_V). \tag{9}$$

In the present case ($N = 4$), this is obtained by computing $\delta(k, x)$ only once. Measuring the content of register X in (9) yields the solution. The quantum network implementing this transformation, as well as the function of register V, can be disregarded here.

Thus, in the original algorithm, the solution is obtained in a deterministic way (for simplicity, we put ourselves in those values of N where the probability of error of Grover's algorithm is zero); relational computation, namely reduction on the solution of the problem under problem-solution interdependence, is completely hidden. However, the random generation of k is not represented physically. We extend the physical representation by adding an ancillary n-qubit register K prepared in a superposition of all the values of k. The extended algorithm repeatedly computes $\delta(k, x)$ as before but now for a superposition of all the combinations of values of k and x. Now the input state is:

$$\begin{aligned}&\frac{1}{4\sqrt{2}}(|00\rangle_K + |01\rangle_K + |10\rangle_K + |11\rangle_K)\\&\quad\times(|00\rangle_X + |01\rangle_X + |10\rangle_X + |11\rangle_X)(|0\rangle_V - |1\rangle_V),\end{aligned} \tag{10}$$

where the superposition hosted in register K can indifferently be coherent or incoherent (in which case each element of the superposition should be multiplied by a random phase). The extended algorithm unitarily transforms the input state (10) into the output state:

$$\frac{1}{2\sqrt{2}}(|00\rangle_K|00\rangle_X + |01\rangle_K|01\rangle_X + |10\rangle_K|10\rangle_X + |11\rangle_K|11\rangle_X)(|0\rangle_V - |1\rangle_V), \tag{11}$$

where each value of k is entangled with the corresponding solution found by the second player (the same value of k but in register X). The final measurement of the contents of

registers K and X in state (11) determines the moves of both players. The reduction induced by measuring the content of register K, backdated to before running the algorithm, yields the original Grover's algorithm. We find reduction on the solution of the problem under problem-solution interdependence.

The nondeterministic production of the contents of the two registers by quantum measurement, can be seen as mutual determination between such contents, like in the measurement of two entangled polarizations. This justifies the square root speed up, as follows. We cannot say that reading the content of K at the end of the algorithm determines the content of X, namely that choosing the drawer to hide the ball in determines the drawer the ball is found in—this would be the classical game with no mutual determination. For the same reason we cannot say that reading the content of X determines the content of K, that looking inside a drawer at the end of the algorithm creates the ball in it. Mutual determination is symmetrical, it should be represented by saying that the contents of the two registers are determined by reading the first (second) bit of register K and the second (first) bit of register X.

Then Grover's algorithm is equivalent to the following game. We arrange the N drawers in a matrix of $\sqrt{N}$ columns and $\sqrt{N}$ rows. At the end of the algorithm, the first player determines (say) the row by reading the first bit of register K. The second player determines the column by reading the second bit of register X, say that this reading is 1. The reduction induced by the second player, backdated to before running the algorithm, changes the initial preparation of register K, $\frac{1}{2}(|00\rangle_K + |01\rangle_K + |10\rangle_K + |11\rangle_K)$ (10), into $\frac{1}{\sqrt{2}}(|01\rangle_K + |11\rangle_K)$, thus determining the column before running the algorithm. In this picture, Grover's algorithm searches just the row randomly chosen by the first player, which justifies the $\sqrt{N}$ computations of $\delta(k, x)$—the quadratic speed up. Grover's algorithm is equivalent to a classical search in a database of size $\sqrt{N}$ (we should symmetrize for the exchange of columns and rows). See also [2, 6, 7].

The same justification holds in the case that the value of k is already determined before running the algorithm, like in virtual database search. This case is indistinguishable from the random generation of k at the end of the algorithm, where backdating reduction makes k predetermined.

If we think that k is predetermined, the initial superposition in register K—$\frac{1}{2}(|00\rangle_K + |01\rangle_K + |10\rangle_K + |11\rangle_K)$—represents the initial ignorance of the value of k on the part of the second player. Its reduction to $\frac{1}{\sqrt{2}}(|01\rangle_K + |11\rangle_K)$, due to the second player reading 1 in the second bit of register X at the end of the algorithm, gives—before running the algorithm—an information gain of one bit. This is 50% of the information acquired by reading the solution, namely the two bits of register either K or X (the information content of one register is redundant with respect to the content of other register), or one bit of K and the other bit of X.

For a database of size N, the reduction of ignorance about the solution due to backdating, to before running the algorithm, 50% of the information acquired by reading the solution, is:

$$\Delta S = \frac{1}{2} \lg_2 N. \tag{12}$$

Summing up, in the relational representation:

1. Quantum computation is *reduction* on the solution of the problem under the relation representing problem-solution interdependence.
2. The speed up is the reduction of the initial ignorance of the solution due to backdating, to before running the algorithm, a time-symmetric part of the reduction on the solution,

namely 50% of the information acquired by reading the solution. This advanced cognition of the solution reduces the size of the solution space to be explored by the algorithm.

3. The quantum algorithm takes the time taken by a classical algorithm (the *reference classical algorithm*) that knows in advance 50% of the information acquired by reading the solution (i.e. by measuring the content of the computer register at the end of the quantum algorithm).

In the present case of Grover's algorithm, backdating 50% of the information about the database location, reduces the size of the solution space (of the database) from N to $\sqrt{N}$. The quantum algorithm working on a database of size N takes the time taken by a classical algorithm working on a database of size $\sqrt{N}$. See also [3–5]. We should note that the formulation of point **3** is independent of the relational representation of computation. Therefore Point **3**, besides being theoretically justified within the relational representation of computation, can be seen as an empirical fact that holds for all quantum algorithms (as clarified here below).

We should note that, in the present context, reduction is on both the problem (the oracle's random choice) and the solution. Therefore, "reduction on the solution of the problem" should be understood as reduction on the solution and the problem. Correspondingly, the information acquired by reading the solution of the problem is the information acquired by reading the solution and the problem. This, in Grover's algorithm, coincides with the information acquired by reading either one, since either one is a function of the other.

5.2 Deutsch's Algorithm

We consider the revisitation of Deutsch algorithm [9] due to [8]. Now the oracle chooses at random one of the four binary functions $f_{\mathbf{k}} : \{0, 1\} \to \{0, 1\}$ (**k** is a two-bit string belonging to $\{0, 1\}^2$); these four functions are reported in table (13)

x	$f_{00}(x)$	$f_{01}(x)$	$f_{10}(x)$	$f_{11}(x)$
0	0	0	1	1
1	0	1	0	1

. (13)

Then the oracle gives to the second player the black box hardwired for the computation of that function. The second player, by trying function evaluation for different values of x, must find out whether the function is balanced (i.e. f_{01} or f_{10}, with an even number of zeroes and ones) or constant (i.e. f_{00} or f_{11}). This requires two function evaluations in the classical case, just one in the quantum case.

In the conventional quantum algorithm, the second player prepares two one-qubit registers X and V in the input state

$$\frac{1}{2}(|0\rangle_X + |1\rangle_X)(|0\rangle_V - |1\rangle_V). \tag{14}$$

With just one function evaluation, the algorithm unitarily produces the output state

$$\frac{1}{\sqrt{2}}|0\rangle_X(|0\rangle_V - |1\rangle_V) \tag{15}$$

if the function is constant and

$$\frac{1}{\sqrt{2}}|1\rangle_X(|0\rangle_V - |1\rangle_V) \tag{16}$$

if the function is balanced. Thus, at the end of the algorithm, register X contains the solution of the problem. The speed up is obtained in a deterministic way, but also in this case the random generation of $\mathbf{k}$ is not represented physically. We extend the physical representation by adding an ancillary two-qubit register K prepared in a superposition (indifferently coherent or incoherent) of all the possible valuations of $\mathbf{k}$. The input state is now:

$$\frac{1}{4}(|00\rangle_K + |01\rangle_K + |10\rangle_K + |11\rangle_K)(|0\rangle_X + |1\rangle_X)(|0\rangle_V - |1\rangle_V). \tag{17}$$

Now the extended algorithm, given the inputs $\mathbf{k}$ and x, computes $f(\mathbf{k}, x) \equiv f_{\mathbf{k}}(x)$, yielding the output:

$$\frac{1}{2\sqrt{2}}[(|00\rangle_K + |11\rangle_K)|0\rangle_X + (|01\rangle_K + |10\rangle_K)|1\rangle_X)](|0\rangle_V - |1\rangle_V). \tag{18}$$

The measurement of the content of registers K and X in the output state determines the moves of both players, as before. The information acquired by reading the contents of registers K and X is 2 bits (the two bits that specify the choice of the first player, the answer of the second player is a function of that choice—which makes the information contained in register X redundant). The information gain due to backdating 50% of the information acquired by reading the content of the two registers is then $\Delta S = 1$ bit. The quantum algorithm takes the time taken by a classical algorithm working on a solution space reduced in size because one bit of information about the solution—namely the value of either $f_{\mathbf{k}}(0)$ or $f_{\mathbf{k}}(1)$—is known in advance. This algorithm must acquire the other bit of information by computing either $f_{\mathbf{k}}(1)$ or $f_{\mathbf{k}}(0)$. Thus the reference classical algorithm performs just one function evaluation like the quantum algorithm. This verifies point **3**.

5.3 Simon's Algorithm

A first player (the oracle) chooses at random a function among the set of the "periodic" functions $f_{\mathbf{k}} : \{0, 1\}^n \to S$, with $S \subseteq \{0, 1\}^n$. The "periodic" function $f_{\mathbf{k}}$, where $\mathbf{k} = k_1, \ldots, k_n$ is a string of Boolean values (excluding the all zeroes string), is such that $f_{\mathbf{k}}(x) = f_{\mathbf{k}}(y)$ if and only if $x = y$ or $x = y \oplus \mathbf{k}$. Here x and y are variables belonging to $\{0, 1\}^n$ and $\oplus$ denotes bitwise addition modulo 2 (see the following example). Then he gives to the second player a black box that, given an input x, computes $f_{\mathbf{k}}(x)$. The second player should find the hidden string $\mathbf{k}$ through function evaluation.

Let us exemplify. With $n = 2$ and $S = \{0, 1\}$, there are three "periodic" functions (up to permutation of function values leaving the hidden string unaltered):

x	$f_{01}(x)$	$f_{10}(x)$	$f_{11}(x)$
00	0	0	0
01	0	1	1
10	1	0	1
11	1	1	0

. (19)

The first player chooses at random a hidden string $\mathbf{k}$, among the $2^n - 1$ possible hidden strings (here 01, 10, and 11), and delivers to the second player the corresponding black box. In order to find $\mathbf{k}$, the second player must perform function evaluation the order of $2^{n/3}$ times in the classical case, the order of n times in the quantum case [21]—there is an exponential speed up.

The original Simon's algorithm is as follows. The second player prepares an n-qubit register X in an even weighted superposition of all the possible values of x, and an m-qubit register F (devoted to contain the result of function evaluation: $m \leq n$) in a sharp state. The input state of the algorithm is thus:

$$\frac{1}{2}(|00\rangle_X + |01\rangle_X + |10\rangle_X + |11\rangle_X)|0\rangle_F. \tag{20}$$

Then he performs function evaluation on the superposition of all the possible values of x, obtaining the intermediate output (say that $\mathbf{k} = 10$, see $f_{10}(x)$ in table (19)):

$$\frac{1}{2}[(|00\rangle_X + |10\rangle_X)|0\rangle_F + (|01\rangle_X + |11\rangle_X)]|1\rangle_F. \tag{21}$$

Now he applies the Hadamard transform (still a unitary transformation) to the state of register X, obtaining the overall output:

$$\frac{1}{2}[(|00\rangle_X + |01\rangle_X)|0\rangle_F + (|00\rangle_X - |01\rangle_X)]|1\rangle_F. \tag{22}$$

In the overall output state, for each value of the function, register X hosts an even weighted superposition of the 2^{n-1} strings $\mathbf{h}_j = h_{j1}h_{j2}\cdots h_{jn}$ orthogonal to $\mathbf{k}$—such that, for all j, $(\sum_{i=1}^{n} h_{ji}k_{ji})$ modulo $2 = 0$; in the example, $\mathbf{h}_1 \equiv 00$ and $\mathbf{h}_2 \equiv 01$ are the two strings orthogonal to $\mathbf{k} \equiv 10$. Note that only the phase of the even weighted amplitudes depend on the value of $f_{\mathbf{k}}(x)$. Therefore, by measuring the content of X in (22), one obtains at random one of the $\mathbf{h}_j$. The entire process (initial preparation of registers X and F, unitary transformation, and measurement of the content of X) is iterated until obtaining $n - 1$ different $\mathbf{h}_j$, which allows to find $\mathbf{k}$ by solving a system of $n - 1$ modulo 2 linear equations.

This formulation of Simon's algorithm leaves the number of iterations of the algorithm unbounded. Alternatively, we can iterate the algorithm a fixed number of times, which leaves a certain probability of failing to find the solution. If the algorithm is iterated, say, $6n$ times, the probability of obtaining $n - 1$ different $\mathbf{h}_j$, thus of finding the solution, is about $\frac{8}{9}$ (e.g. [16]). The computation time taken by the $6n$-iterations Simon's algorithm, in terms of number of elementary logical operations, is the order of n^3.

Now we extend Simon's algorithm to represent the random choice of the hidden string on the part of the oracle. We add an auxiliary n-qubit register K, prepared in an even weighted superposition of the $2^n - 1$ possible valuations of $\mathbf{k}$. The black box that (given input x) computed $f_{\mathbf{k}}(x)$ for a specific $\mathbf{k}$, is now replaced by a black box that, given the inputs $\mathbf{k}$ and x, computes $f(\mathbf{k}, x) \equiv f_{\mathbf{k}}(x)$. The input state is now:

$$\frac{1}{2\sqrt{3}}(|01\rangle_K + |10\rangle_K + |11\rangle_K)(|00\rangle_X + |01\rangle_X + |10\rangle_X + |11\rangle_X)|0\rangle_F, \tag{23}$$

where the superposition hosted in register K is indifferently coherent or incoherent. The overall output state (22) becomes now:

$$+\frac{1}{2}|01\rangle_K[(|00\rangle_X+|10\rangle_X)|0\rangle_F+(|00\rangle_X-|10\rangle_X)|1\rangle_F]$$
$$+\frac{1}{2}|10\rangle_K[(|00\rangle_X+|01\rangle_X)|0\rangle_F+(|00\rangle_X-|01\rangle_X)|1\rangle_F]$$
$$+\frac{1}{2}|11\rangle_K[(|00\rangle_X+|11\rangle_X)|0\rangle_F+(|00\rangle_X-|11\rangle_X)|1\rangle_F]. \quad (24)$$

The first player measures the content of register K in (24), obtaining a state of the form (22). The second player measures the content of register X (thus collecting the first of the $\mathbf{h}_j$) then, leaving register K in its sharp state and working only on registers X and F, iterates up to a total of $6n$ times the original Simon's algorithm (thus collecting the other $6n-1$ valuations of $\mathbf{h}_j$).

In the case that the algorithm finds the solution (with probability $\frac{8}{9}$), there is mutual determination between the content of register K and the content of register X in each of its $6n$ measurements. In more detail: measuring the content of K projects the state of register K on a single valuation of $\mathbf{k}$; which valuation of $\mathbf{k}$ is the result of mutual causality between measuring the content of K and the successive measurements of the content of X. This is completely similar to measuring the polarizations of two photons in an entangled polarization state; the polarization measured first is the result of mutual causality between this measurement and the successive measurement of the second polarization; it is not the case that the first result determines the second, nor that the second determines the first, causality is mutual.

Now, backdating 50% of the information acquired by reading the content of register X (50% of $6n$ readings), reduces the size of the problem not with certainty (like in the case of Grover's and Deutsch's algorithms) but with probability $\frac{2}{3}$—the probability of finding the hidden string with $3n$ readings ($\frac{2}{3}=1-\sqrt{1-\frac{8}{9}}$). Moreover, with this probability, the size of the problem is reduced from 2^n-1 to 1.

The fact that the reference classical algorithm finds the solution implies that the quantum algorithm (whose $\mathbf{h}_j$ comprise those of the reference algorithm) also finds the solution: $\frac{2}{3}$ is thus the probability that both the quantum algorithm and the reference algorithm find the solution. Point **3,** the fact that the quantum algorithm takes the time taken by a reference classical algorithm that knows in advance 50% of the information about the solution**,** applies in a probabilistic way. With probability $\frac{2}{3}$, we put ourselves in the case that both the quantum and the reference algorithm find the solution (the $\frac{1}{3}$ probability that this is not the case, goes down exponentially fast with the number of iterations of the quantum algorithm, it is $\approx(\frac{1}{3})^g$ with $6gn$ iterations). In this case, the reference algorithm has simply to sort out $n-1$ different valuations of $\mathbf{h}_j$ and solve the related system of $n-1$ modulo 2 linear equations. Thus, the quantum algorithm takes a time the order of n^3, the reference algorithm the order of n^2, not so different in comparison with the order of $e^{\frac{n}{3}}$ taken by classical computation.

Perhaps a more accurate way of comparing classical computation and the reference algorithm is to say that:

4. Classical computation has to find the hidden string by solving a system of nonlinear equations in n variables, the reference classical algorithm—thanks to the advanced cognition of 50% of the solution—a system of linear equations in the same order of variables (here $n-1$ variables).

The fact that points **1**, **2**, **3**, and **4** hold for Simon's algorithm implies that they hold for a larger class of problems, because of the similarity between the generalized Simon's problem (essentially solved by Simon's algorithm) and the hidden subgroup problems [17]—like finding orders, finding the period of a function (the quantum part of Shor's factorization algorithm), finding discrete logarithms, etc. [16]. In all cases, utilizing 50% of the information acquired by reading the solution, reduces the probability of finding the solution by an amount that goes exponentially to zero with the number of iterations of the quantum algorithm. Point **4** holds replacing "hidden string " by "hidden subgroup".

6 Relational Computation and the Unity of Perception

For wholeness of perception, as it appears in introspective analysis, I mean the following. For example, in this moment, I see the room in which I am working, an armchair, the window, the garden, and the Mediterranean Sea on the background. In my visual perception, besides some aspects that are addressed by artificial intelligence, like the recognition of patterns, there is another thing that should be addressed by a physical information theory, the both obvious and striking fact that I see so many things together at the same time. What I see is close to a digital picture whose specification would require a significant amount of information. And, apparently, we can perceive a significant amount of information simultaneously all together, in the so called "present". Another example is our capability of grasping the solution of a problem. Reasonably, when we grasp the solution, we should take into account at the same time the statement of the problem, the solution, and the logical connection in between.

In the assumption that perception is information processing, perceiving many things at the same time poses the question: what form of computation can process many things at the same time in the so called "present", and what is the physical counterpart of the introspective notion of "present", such that an entire computation process can occur in it? The mechanism of relational computation can provide an answer.

In the idealized classical case, an entire computation process is condensed into an instantaneous many body interaction. The physical counterpart of the introspective notion of "present" is here the instant of the interaction.

In the quantum case, a state can hold any amount of information, which is processed "at the same time" by the sequence: preparation, unitary transformation, and measurement. "At the same time" since there is simultaneous dependence between all the amplitudes of the computational basis vectors at any pair of times along the process (the change of one amplitude, from the forward to the backward value, changes the other and vice-versa, in a time symmetric way). Correspondingly the measurement interaction changes the entire forward evolution into the backward evolution. The physical counterpart of the notion of "present" is the time interval spanned by backdated reduction.

The observation that, in visual perception, we take into account many things at the same time acquires a literal meaning. Taking into account many things at the same time is exactly what many body interaction, or reduction of the forward evolution on the backward evolution, does.

By the way, if the physical basis of consciousness is a nondeterministic problem solving mechanism, consciousness could not be the passive witness of a deterministic classical process. Moreover, the deterministic, two-body character of classical computation prevents taking into account many (so to speak, more than two) things at the same time, or (reasonably) hosting consciousness either.

By showing that the quantum speed up is advanced cognition of the solution, the present model might provide some theoretical ground to the psychological notion of premonition. For the possibility that consciousness interacts with systems displaced in time up to 500 milliseconds in the past and from milliseconds to months into the future, see [20].

The present identification between the notions of simultaneous dependence, physical law, and perception has a precedent in Plato's notion of Form (the Greek word Eidos translates into Form, Idea, or Vision). In Phaedo: "Ideas are objective perfections that exist in themselves and for themselves, at the same time they are the cause of natural phenomena, they keep phenomena bound together and constitute their unity" [1]. The Ideas of our mind are clearly identified with physical laws; as well known, Platonic Ideas are also perfect mathematical objects. The usual Platonist interpretation of this ambivalence is that the mind can access an autonomous and objective world of perfect mathematical ideas. A more physical interpretation is the other way around, the ideas in our head—our perceptions—are instances of physical laws, namely of objectively perfect, nonfunctional simultaneous dependences.

The present idea that "grasping the solution of a problem" implies reduction under a simultaneous dependence representing problem-solution interdependence, is parallel to another statement of the theory of Forms: "To know the Form of X is to understand the nature of X; so the philosopher who, for example, grasps the Form of justice, knows not merely what acts are just, but also why they are just".

Also Plato's notion that the mind can access the objective perfection of the world of ideas, while material objects are imperfect (a formulation of the mind-body problem) is reflected in the present model, where perception is seen as the instantiation in the brain of an objectively perfect physical law.

The present fundamental problem-solving mechanism matches in my judgement with: (i) the Orchestrated Objective Reduction theory of consciousness of Hameroff and Penrose [15]: it is a way of seeing problem-solving, specifically of a form relying on the non linearity of reduction, in Objective Reduction, (ii) with the idea, also present in that theory, that the very existence of consciousness depends on our capability of accessing the Platonic world of objectively perfect mathematical Ideas [19], and (iii) with Stapp's idea [22] that consciousness is incompatible with classical locality (here seen as two body interaction, unable to process many things at the same time) and compatible with quantum non-locality and reduction (here seen as many body interaction, or reduction under problem-solution interdependence, capable of processing many things at the same time).

7 Conclusions

In the admission of their same authors, the quantum speed ups are still little understood. In his 2001 paper [14], Grover states: "*What is the reason that one would expect that a quantum mechanical scheme could accomplish the search in* $O(\sqrt{N})$ *steps? It would be insightful to have a simple two line argument for this without having to describe the details of the search algorithm.*" The answer provided in this work is summarized in the following points:

1. quantum computation is *reduction* on the solution of the problem under the relation representing problem-solution interdependence;
2. the speed up is the reduction of the initial ignorance of the solution due to backdating, to before running the algorithm, a time-symmetric part of the reduction on the solution; this advanced knowledge of the solution reduces the size of the solution space to be explored by the algorithm;

3. correspondingly the quantum algorithm takes the time taken by a classical algorithm that knows in advance 50% of the information acquired by reading the solution (i.e. by measuring the content of the computer register at the end of the quantum algorithm).

In a situation where heuristics went ahead of theory, and run aground since a few years now, a better understanding of the mechanism of the speed up might refuel theoretical research.

The expounded mechanism could be used, by reverse engineering, for the search of new quantum algorithms. For example, Grover's algorithm could be obtained by symmetrizing (for the exchange of all the possible ways of getting in advance 50% of the information about the solution) a classical algorithm that does database search in a solution space of quadratically reduced size. The fact that the speed up is the reduction of the size of the problem obtained by utilizing 50% of the information acquired by reading the solution, could also be used to investigate which problems are liable of being solved with a quantum speed up, the question of why the speed up is quadratic for unstructured problems, exponential for a very limited class of structured problems, not others, etc.

From another standpoint, relational computation provides a unified vision of disparate forms of computation. One goes from deterministic classical computation to nondeterministic quantum computation by going from two body interaction to many body interaction, and from this to the relational representation of quantum computation. Furthermore, being the condensation of an entire computation process in a single interaction (many body or quantum measurement), this form of computation can represent the information processing standing at the basis of perception. It explains an essential feature of conscious perception, the fact that we see so many things together at the same time in the so called "present". Taking into account many constraints at the same time is exactly what classical many body interaction, or reduction of the forward evolution on the backward evolution, does; the physical counterpart of the introspective notion of "present" is correspondingly the instant of the interaction in the classical case, the time interval spanned by backdated reduction in the quantum case.

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References

1. Abbagnano, N.: Storia della Filosofia, vol. I, p. 87. U.T.E.T., Turin (1958)
2. Castagnoli, G.: Quantum computation based on retarded and advanced propagation. quant-ph/9706019 (1997)
3. Castagnoli, G.: Quantum problem solving as simultaneous computation. arXiv:0710.1744 (2007)
4. Castagnoli, G.: On a fundamental problem solving mechanism explaining the wholeness of perception. In: Hameroff, S.R., Kaszniak, A.W., Scott, A.C. (eds.) Toward a Science of Consciousness 2008—Tucson Discussions and Debates, p. 456. MIT Press, Cambridge (2008)
5. Castagnoli, G.: The mechanism of quantum computation. Int. J. Theor. Phys. (2008). Special issue "Quantum Computation 2008". UV
6. Castagnoli, G., Finkelstein, D.: Theory of the quantum speed up. Proc. R. Soc. Lond. A **457**, 1799 (2001). quant-ph/0010081
7. Castagnoli, G., Monti, D., Sergienko, A.: Performing quantum measurement in suitably entangled states originates the quantum computation speed up. quant-ph/9908015 (1999)
8. Cleve, R., Ekert, A., Macchiavello, C., Mosca, M.: Quantum algorithms revisited. Proc. R. Soc. Lond. A **454**(1969), 339–354 (1996). quant-ph/9708016
9. Deutsch, D.: Quantum theory, the Church-Turing principle and the universal quantum computer. Proc. R. Soc. Lond. A **400**, 97 (1985)

10. Finkelstein, D.R.: Space-time structure in high energy interactions. In: Gudehus, T., Kaiser, G., Perlmutter, A. (eds.) Coral Gables Conference on Fundamental Interactions at High Energy. Center of Theoretical Studies, January 22–24, 1969. University of Miami, pp. 324–343. Gordon & Breach, New York (1969)
11. Finkelstein, D.R.: Generational Quantum Theory. Preprint (2008)
12. Fredkin, E., Toffoli, T.: Conservative logic. Int. J. Theor. Phys. **21**, 219 (1982)
13. Grover, L.K.: A fast quantum mechanical algorithm for database search. In: Proc. 28th Ann. ACM Symp. Theory of Computing (1996)
14. Grover, L.K.: From Schrodinger equation to quantum search algorithm. quant-ph/0109116 (2001)
15. Hameroff, S.R., Penrose, R.: Orchestrated reduction of quantum coherence in brain microtubules: a model for consciousness? In: Hameroff, S.R., Kaszniak, A.W., Scott, A.C. (eds.) Toward a Science of Consciousness—the First Tucson Discussions and Debates, pp. 507–540. MIT Press, Cambridge (1996)
16. Kaye, P., Laflamme, R., Mosca, M.: An Introduction to Quantum Computing. Oxford University Press, New York (2007)
17. Mosca, M., Ekert, A.: The Hidden Subgroup Problem and Eigenvalue Estimation on a Quantum Computer. Lecture Notes in Computer Science, vol. 1509. Springer, Berlin (1999)
18. Mulligan, K., Smith, B.: Mach and Ehrenfels: The foundations of gestalt theory. In: Smith, B. (ed.) Foundations of Gestalt Theory, Munich and Vienna. Philosophia, vol. 124 (1988). http://ontology.buffalo.edu/smith/articles/mach/mach.html-N_1_
19. Penrose, R.: Shadows of the Mind—a Search for the Missing Science of Consciousness. Oxford University Press, London (1994)
20. Sheehan, D.: Consciousness and the physics of time. In: Consciousness Research Abstracts, Proceedings of the Toward a Science of Consciousness Meeting, April 8–12, 2008, Tucson, Arizona
21. Simon, D.: On the power of quantum computation. In: Proc. 35th Ann. Symp. on Foundations of Comp. Sci., pp. 116–123 (1994)
22. Stapp, H.P.: Mind, Matter, and Quantum Mechanics. Springer, Heidelberg (1993)